

## **Analysis of Earthquake Coda Wave for Exploring the Information of the Crust**

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**ABSTRACT:** Attenuation quality factor ( $Q_c$ ) of seismic wave has been analyzed and estimated for coda wave amplitude of digital seismogram which has been recorded at Dhaka University seismic station for the Meiktila, Myanmar earthquake event of  $M$  6.6 taken place at 18:16:13 UTC time on 21<sup>st</sup> September 2003. Frequency dependent quality factor has been determined and shown in this work to study the attenuation. The decay mechanism has been explained by the estimated quality factors and the properties of the local geology through in which the wave propagates. Ten central frequencies starting from 0.25 Hz to 8 Hz are being considered for estimating quality factor using single backscattering model. Frequency dependent quality factor has been obtained as  $Q_c = 9039.127f^{0.919}$ . This obtained quality factor is not strongly dependent on frequency, so it indicates that the area adjacent to the recorded station is seismically and tectonically active and the structure of the crust beneath the area is of less heterogeneous.

**Keywords-**Attenuation, Earthquake, Quality Factor, Seismogram, Seismic Coda wave

### **I. INTRODUCTION**

When an earthquake occurred, seismic wave generates and propagates into earth interior through a medium. Amplitude of this wave gradually decay before the wave diminishes. This phenomenon is called the attenuation of seismic waves and is an important characteristic in the modern seismology which needs to be studied. The attenuation of seismic waves provides important information about the medium characteristics which is required for the determination of earthquake source parameters as well as for prediction of earthquake ground motions. Attenuation of seismic waves is controlled by geometrical spreading, scattering due to inhomogeneities in the medium.

The attenuating property of a medium is described by the dimensionless quantity called quality factor  $Q$ , which expresses the decay of wave amplitude during its propagation in the medium. Quality factor ( $Q_c$ ) of coda waves are estimated to understand the attenuation characteristics. Aki and Chouet [1975] developed the single backscattering model to estimate  $Q_c$ . The estimates of the quality factor have been found to be frequency dependent by several researchers worldwide [2-21]. It is seen as the direct  $S$  wave being followed by wave trains whose amplitude decrease exponentially on seismogram as the lapse time increases. The lapse time is the difference in time between the  $S$  wave starting time and the origin time.

Coda waves from small earthquakes to determine local attenuation properties of the crust [15], [22]. Coda waves from small local earthquakes are the superposition of backscattered body waves generated from numerous heterogeneities distributed randomly in the lithosphere [1], [2], [23]. Therefore, the great variety of paths traveled by these scattered waves provides information concerning the average attenuation properties of the medium instead of just the characteristics of a particular path [15]. According to the tectonic activity and geological age of any region, the  $Q_0$  values are observed to be changed and the frequency dependency of  $Q$  increases with level of tectonic activity.

In the present study, single backscattering model are used to determine the attenuation quality factor ( $Q_c$ ) for getting the crustal information.

**II. METHOD and DATA ANALYSIS**

The single back scattering model are used in this research which developed by Aki (1969) and extended by Aki and Chouet (1975) and Sato (1977) for estimation of coda wave attenuation quality factor,  $Q_c$ .

According to Aki (1969), Aki and Chouet (1975) and Sato (1977), the time dependence of root mean square coda wave amplitude,  $A(\omega, t)$ , on a bandpass-filtered seismogram can be written as:

$$A(\omega, t) = C(\omega) \cdot t^{-1} \exp(-\omega t / 2Q_c) \tag{1}$$

Sato (1977) developed the model where root mean square (rms) coda wave amplitude at lapse time  $t$  may be written as:

$$A(r, \omega, t) = C(\omega) [K(r, x)] \exp(-\omega t / 2Q_c) \tag{2}$$

where,  $x = t/t_s$  ( $t_s$  is the travel time of S wave) and  $r$  is station-source distance;  $K(r, x)$  is a function of  $x$  and  $r$ , defined as:

$$K(r, x) = 1/r \cdot 1/x \cdot \ln(x+1/x-1) \tag{3}$$

By taking the natural logarithms of equation (2) and rearranging terms, we obtain the following equation:

$$\ln [ A(r, \omega, t) / [K(r, x)] ] = \ln [ C(\omega) ] - (\omega t / 2Q_c) \tag{4}$$

For narrow bandpass-filtered seismograms,  $C(\omega)$  is constant. Therefore, by using a linear regression of terms on the left side of equation (4) vs  $t$ ,  $Q_c$  can be determined from the slope of the fit, which is equal to  $\omega t / 2Q_c$ .

**2.1 EARTHQUAKE DATA**

The earthquake event was recorded at Dhaka University seismic station of M 6.6 earthquakes on 21<sup>st</sup> September 2003 of 18:16:13 UTC time. Table 1 lists the source parameters of the selected event. The recorded earthquake seismic waves are shown in Fig. 1.

**Table-1: Earthquake Source parameters :**

Date	Origin time	Location	Depth (Km)	Distance of epicenter	Mw
21 <sup>st</sup> September 2003	18:16:13(UTC) 23:16:13(BST)	19.90N 95.73E	10.0	696.72	6.6

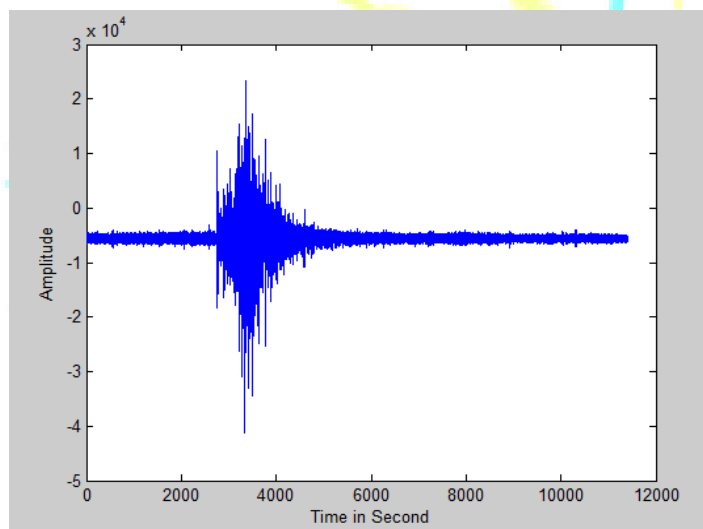


Fig.1). Up-down ground accelerated earthquake seismic wave recorded at Dhaka University of 6.6M earthquake on 21<sup>st</sup> September 2003 at 18:16:13 UTC that occurred at Meiktila, Myanmar.

## 2.2 DATA ANALYSIS

Single backscattering method of Aki & Chouet (1975) is used to estimate the quality factor ( $Q_c$ ). The coda amplitude  $A(f, t)$  in the seismogram for a central frequency 'f' over a narrow bandwidth signal, as a function of the lapse time ( $t$ ), is related to quality factor of coda waves ( $Q_c$ ) by the following equation:

$$A(f, t) = C(f)t^a \exp(-\pi f t / Q_c). \quad (5)$$

The lapse time ( $t$ ) is measured from the origin time of the seismic event and  $Q_c$  represent the average attenuation property of the medium. The factor 'a' represents radiation effect, usually  $a = 1$ , for body wave and  $C = 2(2\Delta f)^{1/2}$ , where  $S$  represents the coda source factors at particular frequency  $f$ . For combining the effects of intrinsic absorption and scattering as:

$$\ln[A(f, t)] = \ln[C(f)] - [\pi f / Q_c]t. \quad (6)$$

Simplifying equation (6) we get

$$\ln[A(f, t)] = c - bt \quad (7)$$

Where,  $b = \pi f / Q_c$  and  $c = \ln C(f)$ .

Eq. (7) represents a straight line and its slope ( $b = \pi f / Q_c$ ) facilitate computation of  $Q_c$  at the central frequency  $f$ .

To find the dependence of  $Q_c$  on frequency the procedure is applied to seismograms band pass filtered at different center frequencies  $f_m$ . The low and high frequency cut offs of the filtered signal  $A(f_m, t)$  are  $(f_m - f_m/3)$  and  $(f_m + f_m/3)$  respectively. The root mean square amplitude (RMS) of  $A(f_m, t)$  are calculated by sliding windows:

$$A(f_m, t_n) = [A_i^2 / (l + 1)]^{1/2} \quad (8)$$

Where  $t_n$  is the central time of the  $n$ th window and  $l$  is the number of data point

When the source and receiver are not coincident, the source receiver distance  $r$  must be taken into account [12].

Then the RMS coda wave amplitude can be described as:

$$A(f, t) = C(f)k(r, x) \exp[-\pi f t / Q_c] \quad (9)$$

Where  $x = t / t_s$ ,  $t_s$  is  $S$ -wave lags time

$$K(r, x) = (1/r) \{ (1/x) \ln[(x+1)/(x-1)] \} \quad (10)$$

Taking natural logarithms of equation (5)

$$\ln[A(f, t) / K(r, x)] = \ln[C(f)] - [\pi f / Q_c]t \quad (11)$$

For narrow band pass filtered seismograms  $C(f)$  is a constant and  $Q_c(f)$  can be determined by applying exactly the same procedure as that described for the case of the collocated source and receiver.

### III. ESTIMATION of $Q_c$ VALUES

We used the waveforms of a regional earthquake  $M$  6.6, epicentral distance 696.72 km and focal depth 10 km recorded at Dhaka University seismic station (Fig. 1).

Coda waves of above mentioned regional earthquake was analysed at central frequencies of 0.25, 0.5, 0.75, 1.5, 2, 3, 4, 6, and 8 Hz for lapse time window length of 30 s. The beginning of this window in each seismogram was taken at time  $2t_s$ , where  $t_s$  is the travel time of direct  $S$  wave. This is necessary to avoid direct  $S$ -wave arrivals [2]. The seismograms were filtered at ten different frequencies using Butterworth band pass filter (eight poles), at different frequency bands (Table 2). The coda for all the filtered seismograms were smoothed calculating the RMS values of coda amplitude of the filtered seismograms with a sliding window of 5 s length in steps of half the window length. These RMS values constitute a smoother envelope of the coda. The  $Q_c$  values (Table 2) were estimated from the coda decay slope with respect to lapse time for the best fit line to the amplitude of records filtered at the central frequency.

The coda wave amplitude at central frequency  $f$  and elapsed time  $t$  from the origin ( $A(f, t)$ ) is found by band pass filtering the coda window trace data using a 8-pole Butterworth filter centered at frequency  $f$  and calculating RMS values using a sliding window of length  $7/f$  sec. Then a time decay envelope is fitted to this filtered signal. Finally, values of  $Q_c$  are calculated using the slope of the linear regression of the logarithm of product of RMS amplitude and lapse time  $\ln(A(f, t))$  against lapse time  $t$ . The slope of such graph,  $b$  is given by  $Q(f) = \frac{\pi f}{b}$  and so values of  $Q_c$  can be calculated. The  $Q_c$  values for such data sets were then averaged for each central frequency. Also the standard deviation for each central frequency was calculated.

**Table-2: Parameters of band-pass filter showing central frequencies with respective low and high cut off frequencies.**

Low cutoff (Hz)	Central frequency(Hz)	High cutoff (Hz)
0.15	0.25	0.35
0.35	0.5	0.65
0.5	0.75	1
0.65	1.0	1.35
0.9	1.5	2.10
1.35	2	2.65
1.90	3	4.1
2.65	4	5.35
4	6	8.0
6.30	8	9.7

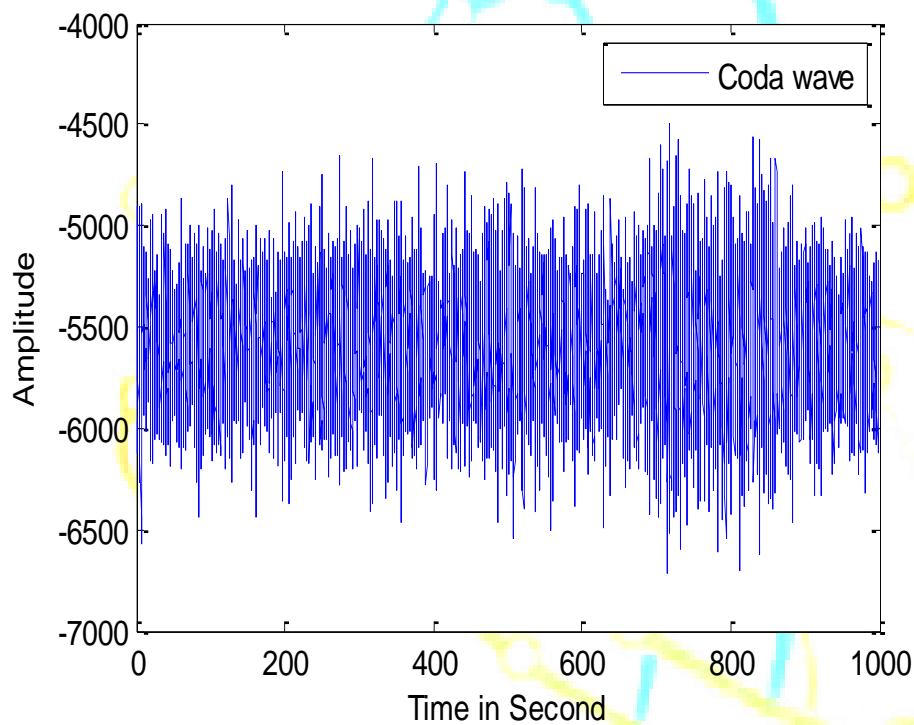
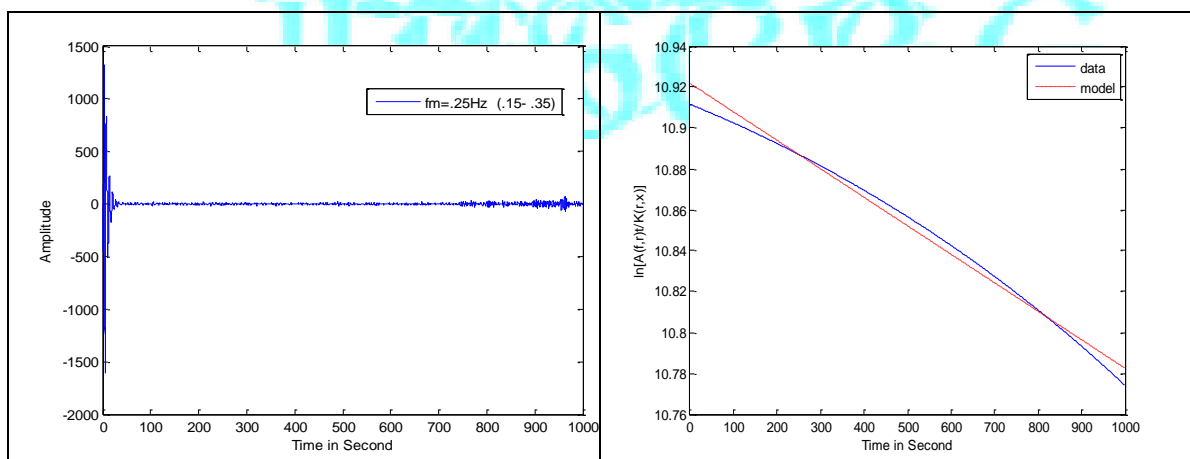
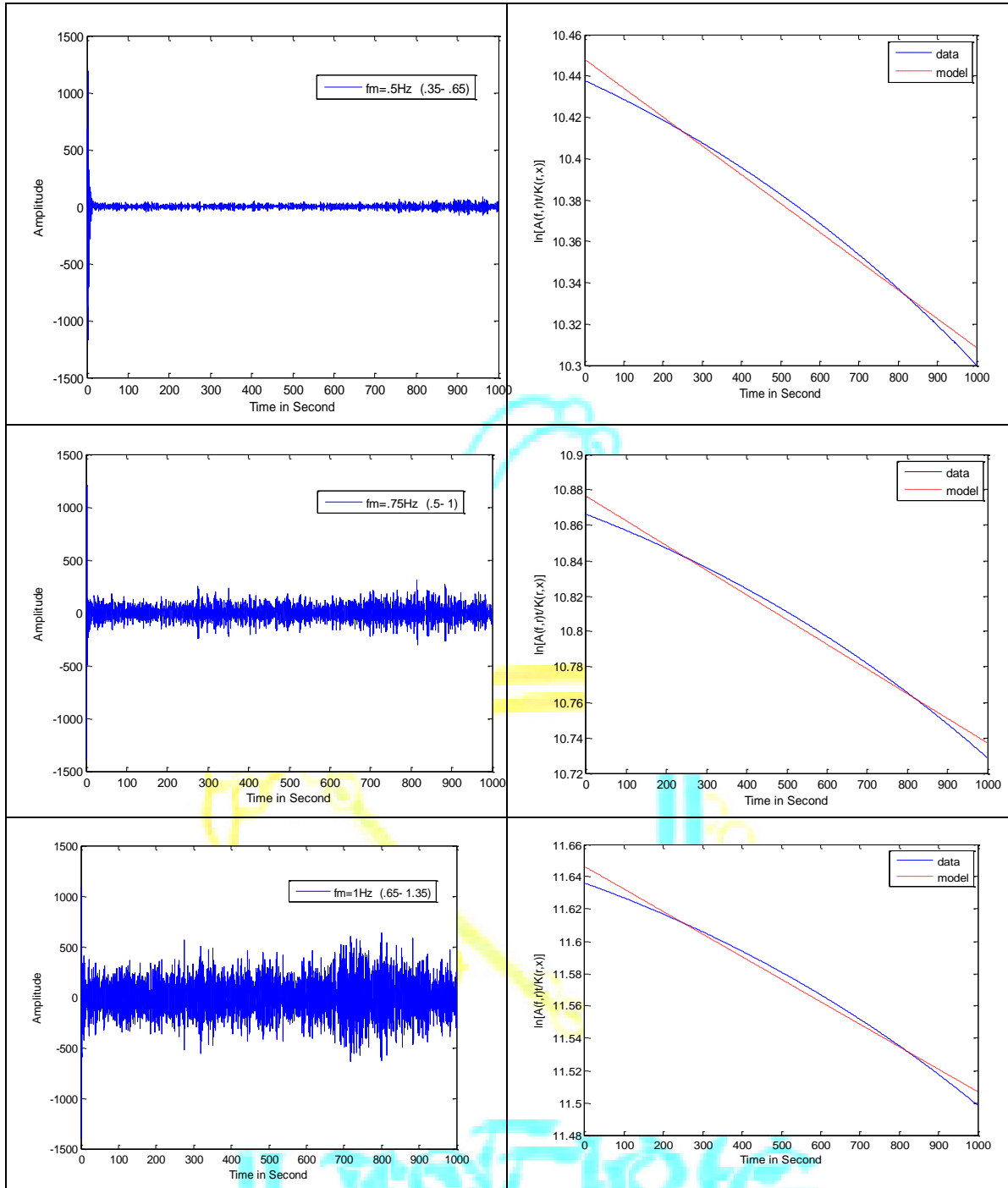
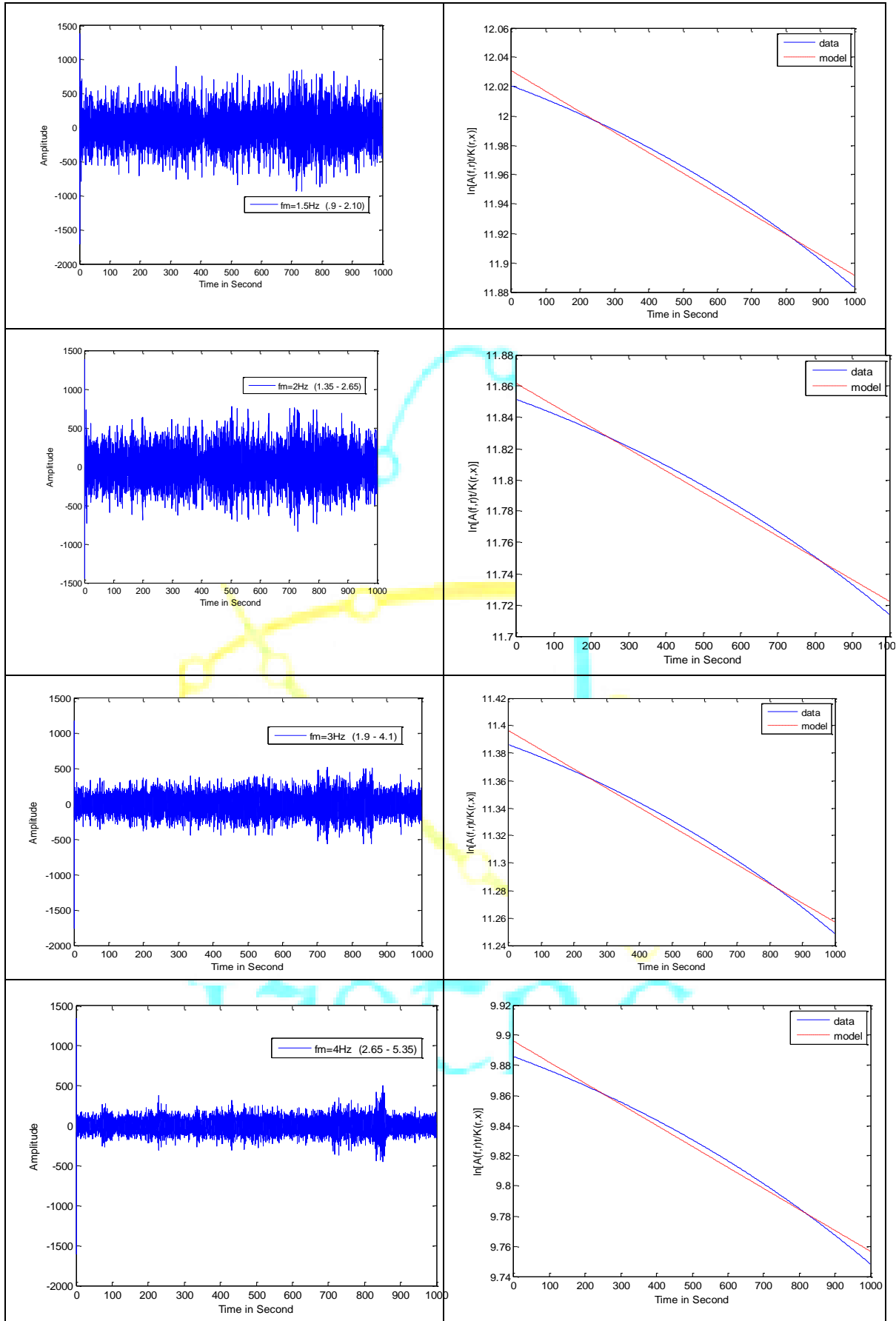


Fig.2. Up-down ground accelerated earthquake seismic wave recorded at Dhaka University of 6.6M earthquake on 21<sup>st</sup> September 2003 at 18:16:13 UTC that occurred at Meiktila Myanmar.





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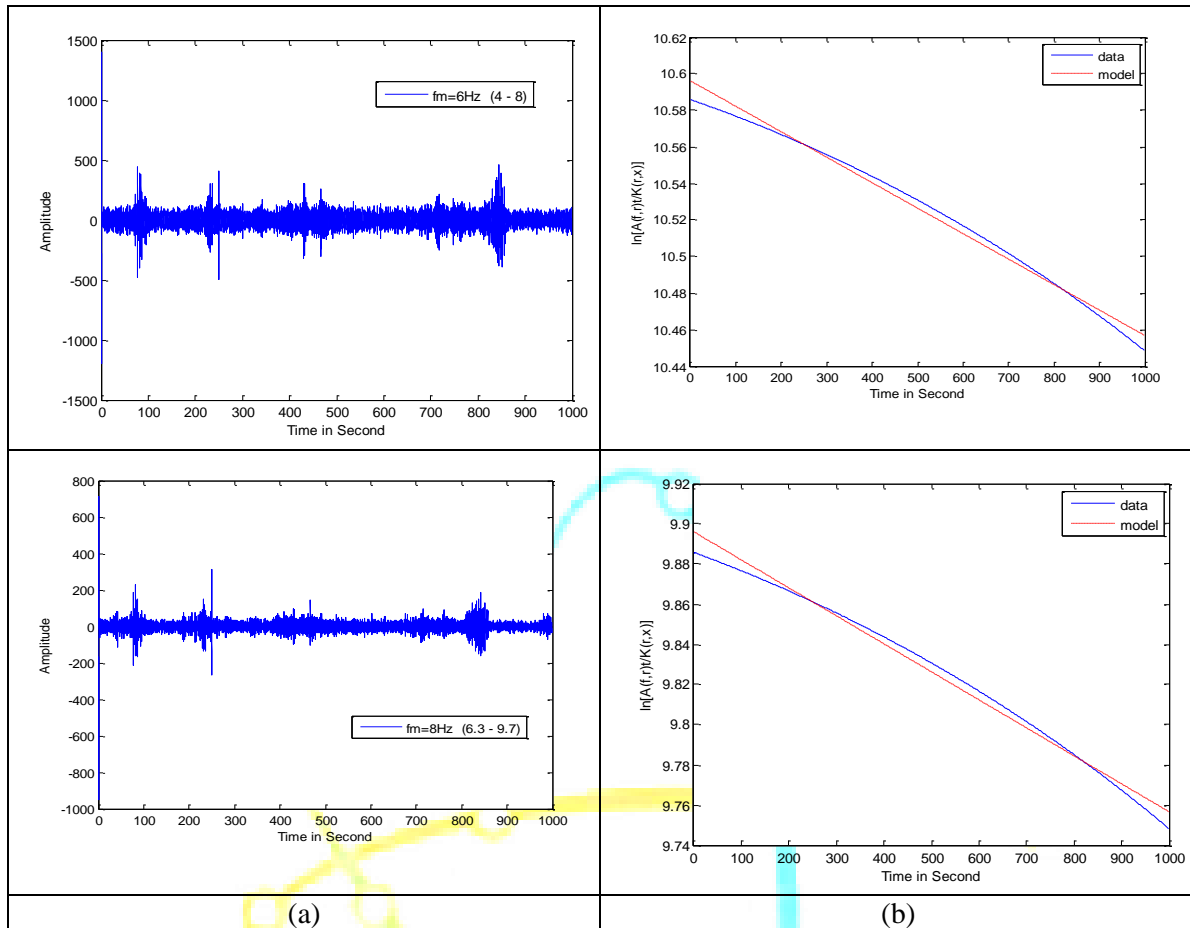


Fig.3. (a) Band pass filtered seismogram and (b) Shows the plot of  $\ln[A(r,f,t)/K(r,x)]$  vs  $t$

#### IV. ESTIMATION of FREQUENCY DEPENDENT RELATIONSHIPS

The  $Q$  factor increases with frequency (Mitchell, 1981) and it follows the following relation:

$$Q_c = Q_0 f^\alpha$$

Where  $Q_0$  is the quality factor at the reference frequency  $f_0$  (generally 1Hz) and  $\alpha$  is the frequency parameter.  $Q_0$  changes with  $\alpha$  as the region varies due to the heterogeneity of the medium [4]. This relation shows that attenuation as the wave propagates is different for different frequencies. Hence seismic data are first band pass filtered when calculations of attenuation are made.

Frequency dependent relationship of  $Q_c$  values of 0.25, 0.5, 0.75, 1, 1.5, 2, 3, 4, 6 and 8 Hz for lapse time of total 1000 sec.  $Q_c$  measurements are obtained for laps time, estimated from the liner regression of  $\ln[A(r,f,t)/K(r,x)]$  vs  $t$  plot. The  $Q_c$  values for a laps time and frequency ranges are listed in Table-3

Table-3:  $Q_c$  Values for laps time of the event

Frequency in Hz	Total 1000 sec
0.25	2618
0.50	3927
0.75	6543
1.0	9163
1.50	15708
2	17017
3	28798
4	35343
6	52360
8	44506

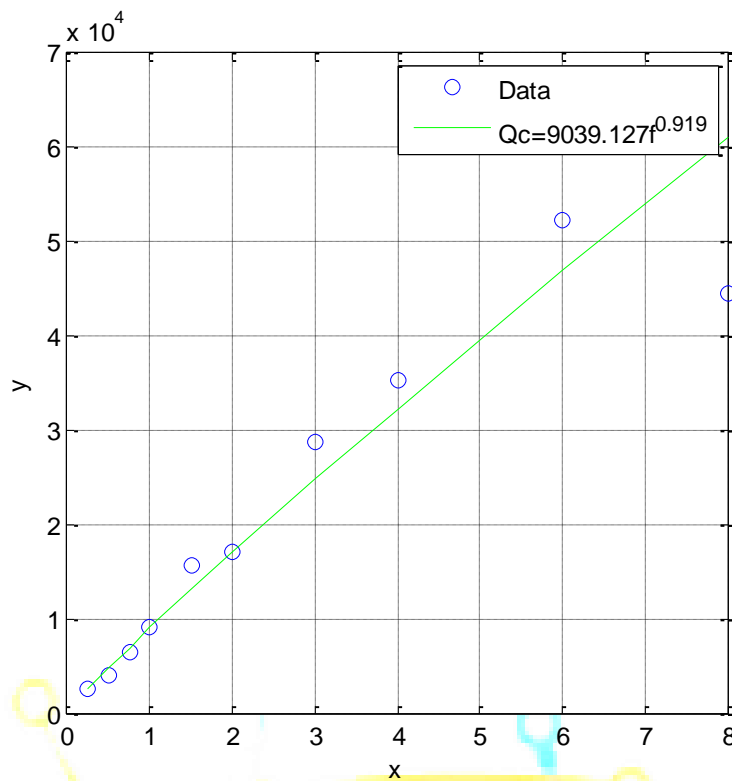


Fig.4. Distribution of  $Q_c$  values with frequency for total 1000 sec.

## V. RESULT and DISCUSSION

The estimation of  $Q_c$  values that are filtering from the coda waves of a waveform in frequency band centered at 0.25, 0.5, 0.75, 1, 1.5, 2, 3, 4, 6 and 8 Hz for lapse time window length of total 1000 seconds. Total 10  $Q_c$  measurements are obtained for total 1000 seconds coda window length estimated from the linear regression of the  $\ln[A(f, t)/K(r, x)]$  vs  $t$  plot. The  $Q_c$  values for all lapse time and frequency ranges are listed in Table-3. The distribution of  $Q_c$  values with frequency are shown in Fig-4.

It is observed from the general trend (Fig. 4) that  $Q_c$  values follows a power law of the form  $Q_c = Q_0 f^n$ , where  $Q_0$  is the quality factor at 1Hz and  $n$  is the frequency dependent coefficient. For total 1000 seconds coda window  $Q_0$  and  $n$  are 9039.127 and 0.919 respectively and we obtain the frequency dependent attenuation relation as  $Q_c = 9039.127 f^{0.919}$ .

From the above results it is observed that  $Q_c$  values obtained for the seismograms are highly frequency dependent. The  $Q_c$  values increases with increase in frequency. The high frequency dependent characteristics of the  $Q_c$  values may be due to different heterogeneity present in the propagating media. Aki(1980) observed that only highly fractured media can generate frequency dependent  $Q_c$  values. Moreover it is observed that  $Q_c$  value increases with increase in coda window length. The variation  $Q_c$  values with lapse time is plot-ted in Figure 4. The degree of frequency dependence 'n' is almost constant whereas  $Q_0$  values are increasing with increase in lapse time coda window length. The higher lapse time of coda window samples larger area of the earth's crust covering the deeper part. The  $Q_c$  value increases with depth implies that attenuation is decreasing with depth. This may be due to the fact that homogeneity increases with depth.

## VI. CONCLUSION

It is seen from this study,  $Q_c$  values are frequency dependence of the propagation media. It is found that there  $Q_0$  value of 9039.127 and corresponding  $n$  value of 0.919 from the frequency dependent relationship. Therefore, the region is intermediate active and less heterogeneous. The value of frequency dependence  $n$  ( $> 1$ ) suggests that the media is not homogeneous. Since the media is non-homogeneities, intrinsic and scattering mechanism takes place causing the attenuation of the coda wave amplitudes. Attenuation decreases with depth may indicating that deeper part of the earth's crust is comparatively homogeneous than the upper most crust.



## VII. ACKNOWLEDGEMENT

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