MAGNETOHYDRODYNAMIC FLOW OF VISCOUS ELECTRICALLY CONDUCTING INCOMPRESSIBLE FLUID THROUGH VERTICAL PLATES SUBJECTED TO INCLINED MAGNETIC FIELD.

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ABSTRACT : *Magnetohydrodynamic (MHD) flow of viscous electrically conducting incompressible fluid through vertical plates has been studied in this paper. We have investigated the steady incompressible viscous fluid flow along y-axis through the vertical plates distanced from x = -D to x =D, the plate at x = -D is stationary while the plate at x= D is moving and applied the inclination angles of magnetic field to the fluid flow. MHD flows find application in geophysics, astrophysics. Engineers employ MHD principles in the design of heat exchangers, pumps and flow meters. The equations of the fluid flow have been subjected to dimensional analysis and are solved using analytical approaches with the help of boundary conditions. The findings are based on the effects of Hartmann number, angle of inclination, pressure gradient and gravitational force and Reynolds number and are presented graphically. It has been found that decreasing the negative values of Hartmann numbers leads to increase of velocity profile, the increase of angle of inclination leads to an increase of velocity profile, decreasing the negative value of Q leads to increase of velocity distribution and finally the increase of the Reynolds number leads to increase of velocity distribution.*

KEYWORDS: *Angle of inclination, Hartmann number, Magnetohydrodynamic flow, Reynolds number and Q (pressure gradient and gravitational force).*

I. INTRODUCTION

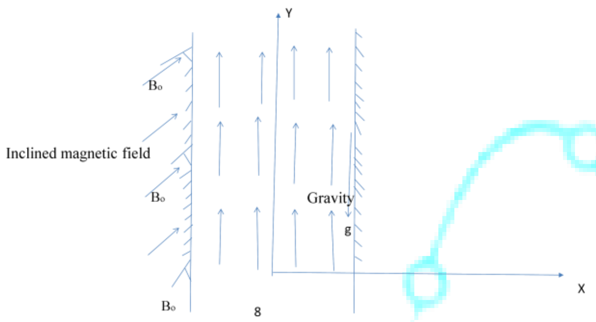
Magnetofluiddynamics is the study of the flow of electrically conducting fluids in the presence of magnetic field. It unifies in a common framework the electromagnetic and fluid dynamic theories to yield a description of the concurrent effects of the magnetic field on the flow and the flow on the magnetic field. Magnetofluiddynamics (MFD) deals with an electrically conducting fluid, whereas its subtopics, magnetohydrodynamics (MHD) and magnetogasdynamics (MGD) are especially concerned with electrically conducting liquids and ionized compressible gases respectively.

MHD phenomena result from the mutual effect of a magnetic field and conducting fluid flow across it. An electromagnetic force is produced in the fluid flowing across a transverse magnetic field and the resulting current and magnetic field combine to produce a force that resists the fluid motion. The current also generates its own magnetic field which distorts the original magnetic field. An opposing, or pumping force on the fluid can be produced by applying an electric field perpendicularly to the magnetic field [1] [2].

Some researchers have done on MHD flows using different channels; Soundalgekar (1979) solved MHD free convective flow at vertical plate using Laplace –transform technique. They found that increasing magnetic field increases the velocity in heated plate while decrease the velocity in the cooled plate. If the plate is cooled in a great low temperature, the velocity increases [3]. Raptis and Singh (1983) investigated the accelerated vertical plate and used the Laplace –Transform Technique and discovered that the skin friction decreases due to the effects of magnetic parameter. And the increase of magnetic forces decreases the velocity field [4]. Helmy (1998) studied MHD free convection past vertical porous plate using perturbation technique. He found that increasing magnetic parameter affects velocity and temperature profile decreases [5]. Hazarika (2011) studied the effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence of magnetic field and found the effects of magnetic field on the velocity and temperature profiles. An increase of magnetic field strength decreases the velocity and the temperature of heated plate and increase the temperature of cooled plate [6]. R. Lakshmi (2014) studied the numerical solution of MHD flow over a moving vertical porous plate with heat and mass transfer and solved the equations using Runge-Kutta fourth order integration scheme together with shooting method and he found that increasing Prandtl number results in decreasing the velocity field. He also discovered that an increase in the radiation parameter and heat source results an increase in the velocity with the boundary layer, also it increases the thickness of the boundary layer. It was observed that as the chemical reaction parameter increases, the velocity profiles decreases. The temperature profiles increase with an increase of magnetic field parameter which implies that the applied magnetic field tends to heat the fluid, and thus reduces the heat transfer from the wall [7]. Nor Raiham Mohamad Asimoni et al (2017) studied MHD free convective flow in incompressible viscous fluid past a vertical plate in the presence of magnetic field and found that an increase of magnetic field strength will decrease the velocity of fluid flow. He also found that the strength of magnetic field will increase the temperature for cooled plate and decrease the temperature for heated plate [8]. Mburu (2016) investigated MHD flow past parallel plates and found that the increase of Hartmann number will decrease the velocity, when the angle of inclination is small then the velocity is maximum. As the angle of inclination increases then the velocity decreases. They also found that the increase in pressure gradient leads to increase in velocity and the increase in Re leads to decrease in velocity and a decrease in Re leads to increase in velocity [9]. Thiele (1999) studied MHD fluid flow problems involving spatially varying viscosity and investigated its effect on Fluid velocity and Magnetic field. And they discovered that the velocity profile is decreased due to high effect of viscosity [10].

The steady MHD flow, viscous, incompressible, electrically conducting fluid along heat and mass transfer for vertical plates in the presence of the magnetic field has been investigated by a number of researchers. The researchers have investigated the incompressible fluid flow past vertical or porous plates with heat and mass transfer and some chemical reaction under transverse normal magnetic field.

We want to investigate the steady incompressible viscous fluid flow along y-axis between two vertical plates distanced from x = -D to x =D, the plate at x = -D is stationary while the plate at x= D is moving and apply the inclination angles of magnetic field to the fluid flow. The study of the MHD flow problems of electrically conducting fluids is currently receiving considerable interest. For example, because of the continuum the universe is filled with widely spaced, charged particles and permeated by magnetic fields and so the continuum assumption becomes applicable. Again, in control and re-entry problems, in designing communications and radar systems; in developing confinement schemes for controlled fusion.



II. EQUATIONS GOVERNING THE FLUID FLOW

2.1 Equation of Motion

$$\frac{\partial \rho}{\partial t}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (1)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu(\nabla^2 u)+\rho g_x+F_X \quad (2)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu(\nabla^2 v)+\rho g_y+F_y \quad (3)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu(\nabla^2 w)+\rho g_z+F_z \quad (4)$$

Where

$\nabla^2=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$And F_x=J X B F_y=J X B F_z=J X B

2.2 Electromagnetic equations

These are equations that give the relationship between electric field intensity E, magnetic induction vector B, electric displacement D, magnetic field density H, induced current density vector J and the charge density p.

If the current J is passing through a conductor under a magnetic flux B, then the conductor experiences a force perpendicular to both of them and is proportional to their magnitude. The force F is called electromagnetic force or Lorentz force and is given by F=J X B

2.2.1. Maxwell's equations

Maxwell's equation is a set of four equations that describe the properties of electric field and magnetic field and relate them to their sources, charge density and current density

$$\frac{\partial B}{\partial t} = -\nabla X E or \frac{\partial H}{\partial t} = -\frac{\nabla X E}{\mu_e} where B = \mu_e \mathrm{H} \quad (5)$$

$$\nabla X H = J \quad (6)$$

$$\nabla . B = 0 \quad (7)$$

$$\nabla \cdot D = \rho_e \quad (8)$$

2.2.2. Ohm's Law

The ideal ohm's law neglects contribution to E for resistivity. The ideal Ohm's law leads to the magnetic field and plasma being frozen into each other so the magnetic topology is preserved.

$$J = \sigma (E + VXB) \quad (9)$$

2.2.3. Induction Equation

Using the generalized Ohm's Law and the Maxwell's equation we obtain the induction equation.

$$\frac{\partial H}{\partial t} = \nu \nabla^2 \mathrm{H} + \mathrm{curl}\,(\mathrm{VXH}) \quad (10)$$

III. SOLUTIONS OF GOVERNING EQUATIONS

The velocity components u= (0, u, 0)

The magnetic field components B= $(B_0 \sin\beta, 0, 0)$

Force F= JXB where J=σ (VXB)

$$J=\sigma (VXB) \quad J=\sigma \begin{pmatrix} i & j & k \\ 0 & u & 0 \\ B\sin\beta & 0 & 0 \end{pmatrix} = -(\sigma B \sin\beta) u \boldsymbol{k}$$

$$JXB = \begin{bmatrix} i & j & k \\ 0 & 0 & -\sigma B \sin\beta u \\ B\sin\beta & 0 & 0 \end{bmatrix} = (\sigma {B^2}_0 \sin^2\beta)\, u \boldsymbol{J}$$

Equations (1), (2), (3) and (4) reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (11)$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = -\frac{\partial p}{\partial x} + \mu[\nabla^2 u] + \rho g_x + (\sigma B^2 \sin^2\beta)\, u \quad (12)$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right] = -\frac{\partial p}{\partial y} + \mu[\nabla^2 u] + \rho g_y + (\sigma B^2 \sin^2\beta) u \quad (13)$$

$$\rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right] = -\frac{\partial p}{\partial z} + \mu[\nabla^2 u] + \rho g_z + (\sigma B^2 \sin^2\beta)u \tag{14}$$

Non-Dimensionalizing the above governing equations using

$$u = Uu', v = Uv', w = Uw', x = Dx', y = Dy', z = Dz', \quad t = \frac{D}{U}t', \quad p = \rho U^2 p', Re = \frac{UD}{\nu}$$

Where D is the characteristic distance. We get

$$\frac{U}{D}\left(\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial z'}\right) = 0 \tag{15}$$

$$\frac{U^2}{D}\left(\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} + w'\frac{\partial w'}{\partial z'}\right) = \frac{U^2}{D}\left(-\frac{\partial p'}{\partial x'} + \frac{\nu}{DU}\{\nabla^2 u'\} + \frac{(\sigma DB^2 \sin^2\beta)u'}{U\rho}\right) + g_x \tag{16}$$

$$\frac{U^2}{D}\left(\frac{\partial v'}{\partial t'} + u'\frac{\partial v'}{\partial x'} + v'\frac{\partial v'}{\partial y'} + w'\frac{\partial w'}{\partial z'}\right) = \frac{U^2}{D}\left(-\frac{\partial p'}{\partial y'} + \frac{\nu}{DU}\{\nabla^2 v'\} + \frac{(\sigma DB^2 \sin^2\beta)v'}{U\rho}\right) + g_y \tag{17}$$

$$\frac{U^2}{D}\left(\frac{\partial w'}{\partial t'} + u'\frac{\partial w'}{\partial x'} + v'\frac{\partial w'}{\partial y'} + w'\frac{\partial w'}{\partial z'}\right) = \frac{U^2}{D}\left(-\frac{\partial p'}{\partial z'} + \frac{\nu}{DU}\{\nabla^2 w'\} + \frac{(\sigma DB^2 \sin^2\beta)w'}{U\rho}\right) + g_z \tag{18}$$

Removing the dashes from equations (15), (16), (17) and (18),

Again the fluid flow is two dimensional $\partial w/\partial z = 0$

The fluid flow is steady $\frac{\partial}{\partial t} = 0$

The fluid flow is in y-direction meaning $u = w = 0$

For the fluid flows which have the vertical slope, the gravity is considered to be $\alpha h = g\sin\beta$, *the change in gravity* $-\alpha dh = g\sin\beta\, dy$

Reynolds number Re=$\frac{UD}{\nu}$

Putting the above conditions in equations (15), (16), (17) and (18), we obtain

$$\frac{\partial u}{\partial y} = 0 \tag{19}$$

$$0 = -\frac{dp}{dx} \tag{20}$$

$$0 = -\frac{dp}{dy} - \alpha\frac{dh}{dy} + \frac{1}{Re}\frac{d^2u}{dx^2} + \frac{(\sigma DB^2 \sin^2\beta)u}{U\rho} \tag{21}$$

Equation (20) shows that the pressure is independent on x meaning that the pressure depends only on y.

Equation (21) can be re-written as

$$\frac{d(p+\alpha h)}{dy} = \frac{1}{Re}\frac{d^2u}{dx^2} + \frac{(\sigma DB^2 \sin^2\beta)u}{U\rho} \quad (22)$$

Differentiating equation (22) with respect to y, we get

$$\frac{d^2(p+\alpha h)}{dy^2} = 0r\,\frac{d}{dy}\left(\frac{d(p+\alpha h)}{dy}\right) = 0 \quad (23)$$

Integrating (23), we obtain

$$\frac{d(p+\alpha h)}{dy} = constant = Q\ (say) \quad (24)$$

Substituting (24) into (22) we get

$$\frac{d^2u}{dx^2} + \frac{(\sigma D^2B^2\sin^2\beta)u}{\mu} = QRe \quad (25)$$

But Hartmann number M= $DB\sqrt{\frac{\sigma}{\mu}}$

Substituting Hartman number in (25) we obtain

$$\frac{d^2u}{dx^2} + (M^2\sin^2\beta)u = QRe \quad (26)$$

Equation (26) is non-homogeneous second ordinary differential equation. To solve equation (26) we first solve for homogeneous part of it and we solve also for the particular integral and we combine two solution.

3.1. Solution for Homogeneous part u_c

$$D^2 + M^2\sin^2\beta = 0\,, D = \pm M\sin\beta i$$

$$u_c = A\cos(M\sin\beta)x + B\sin(M\sin B)x \quad (27)$$

3.2. Solution for Particular Integral

$$u_p = \frac{QRe}{(D^2+M^2\sin^2\beta)}(\cos ax + sinax) = \frac{QRe}{M^2\sin^2\beta}\, but\ a = 0 \quad (28)$$

Combining (27) and (28) we get the general solution for velocity distribution which is expressed in terms of Hartmann number, Reynolds number and Pressure and Gravitational forces

$$\boldsymbol{u = A\cos(M\sin\beta)x + B\sin(M\sin\beta)x + \frac{QRe}{M^2\sin^2\beta}} \quad \mathbf{(29)}$$

Where A and B are constants to be determined using the boundary conditions.

$$u = 0\ when\ x = -1\ and\ u = 1\ when\ x = 1 \quad (30)$$

Using the boundary condition (30), (29) becomes

$$0 = A\cos(M\sin\beta) - B\sin(M\sin\beta) + \frac{QRe}{M^2\sin^2\beta} \tag{31}$$

$$1 = A\cos(M\sin B) + B\sin(M\sin\beta) + \frac{QRe}{M^2\sin^2\beta} \tag{32}$$

Adding (31) and (32) we get

$$A = \frac{M^2\sin^2\beta - 2QRe}{2M^2\sin^2\beta\cos(M\sin\beta)} \tag{33}$$

Subtracting (32) from (31) we obtain

$$B = \frac{1}{2\sin(M\sin\beta)} \tag{34}$$

Using (33) and (34) equation (29) reduces to

$$u = \frac{M^2\sin^2\beta - 2QRe}{2M^2\sin^2\beta\cos(M\sin\beta)}\cos(M\sin\beta)x + \frac{1}{2\sin(M\sin\beta)}\sin(M\sin\beta)x + \frac{QRe}{M^2\sin^2\beta} \tag{35}$$

OR

$$u = \frac{\sin M(\sin\beta x + \sin\beta)}{\sin(2M\sin\beta)} - \frac{QRe}{M^2\sin^2\beta}\left(\frac{\cos(M\sin\beta)x}{\cos(M\sin\beta)} - 1\right) \tag{36}$$

IV. RESULTS

The Navier-Stokes equations have been solved analytically in order to evaluate the fluid flow and the results were plotted using MATLAB. The analysis of the results were based on the different parameters used in this work that is the Hartmann number, the angle of inclination, the pressure gradient and gravitational force and the Reynolds number . The graphs resulted from analyzing each parameter and keeping the others fixed. The upstream velocity is considered to be U =1.

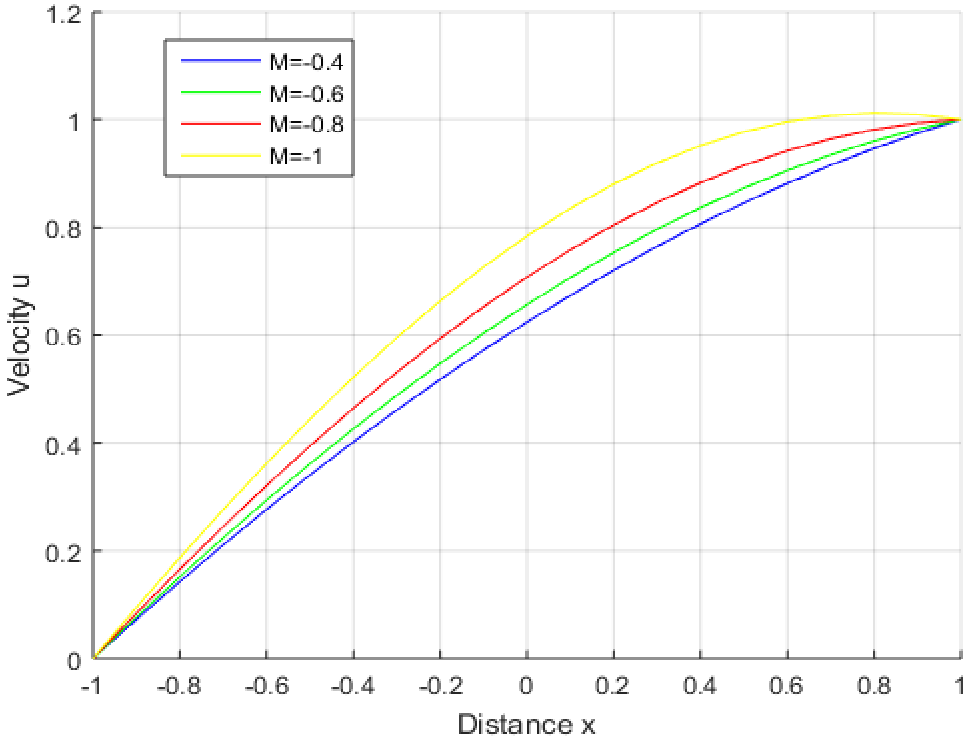

M=-0.4
M=-0.6
M=-0.8
M=-1
Velocity u
Distance x
1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Fig 4. 1

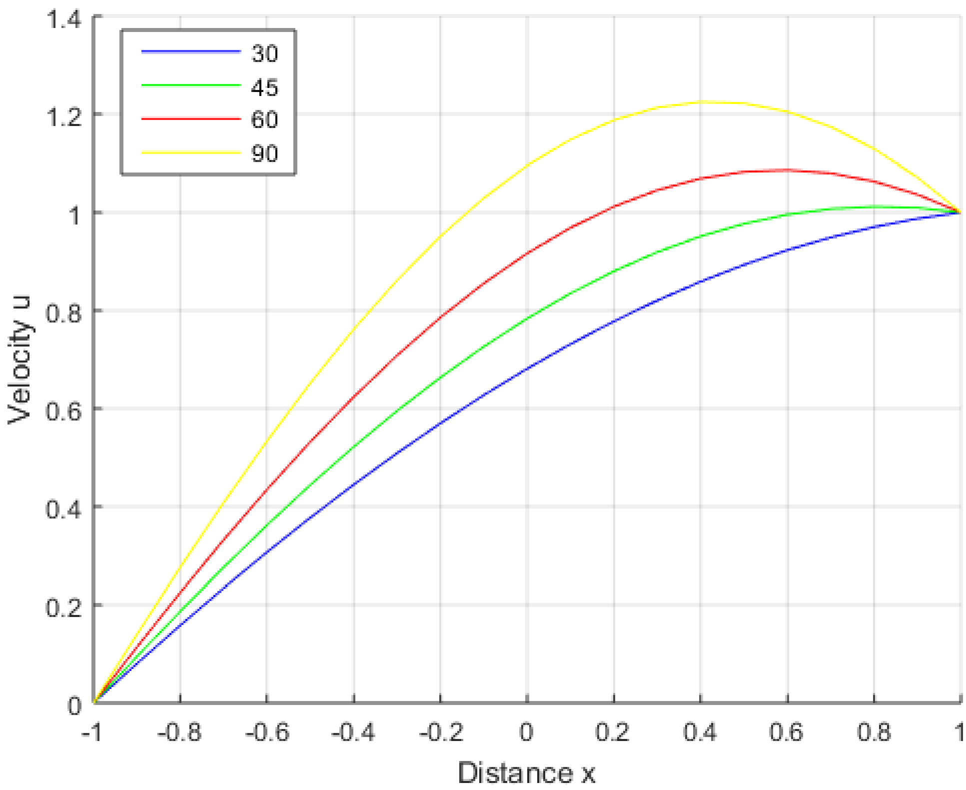

30
45
60
90
Velocity u
Distance x
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Fig 4. 2

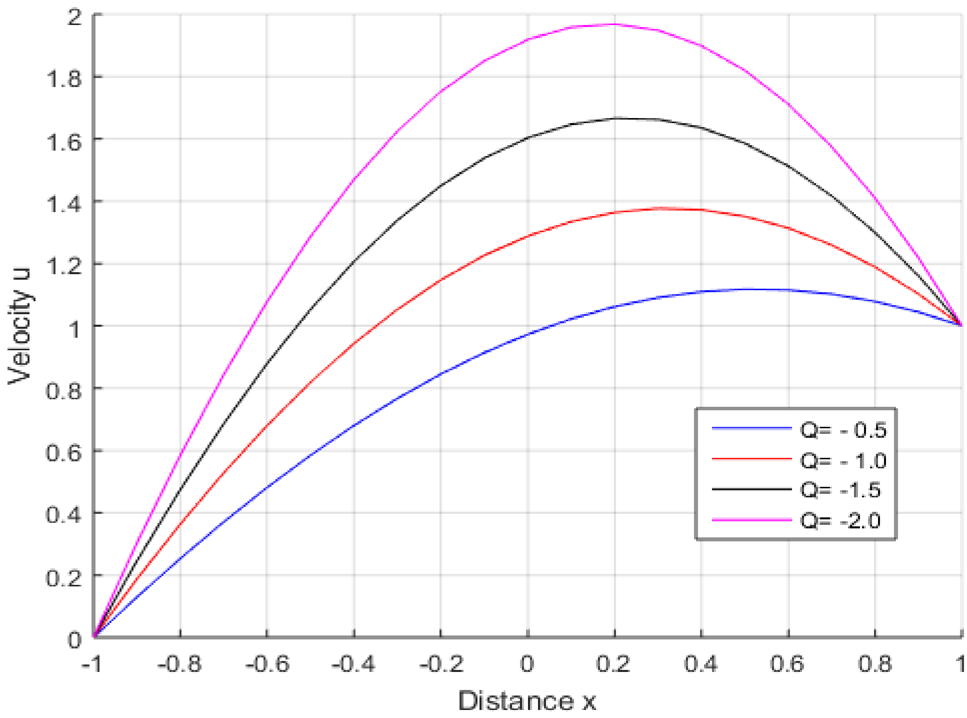

Velocity u
Distance x
Q= - 0.5
Q= - 1.0
Q= -1.5
Q= -2.0

Fig 4. 3

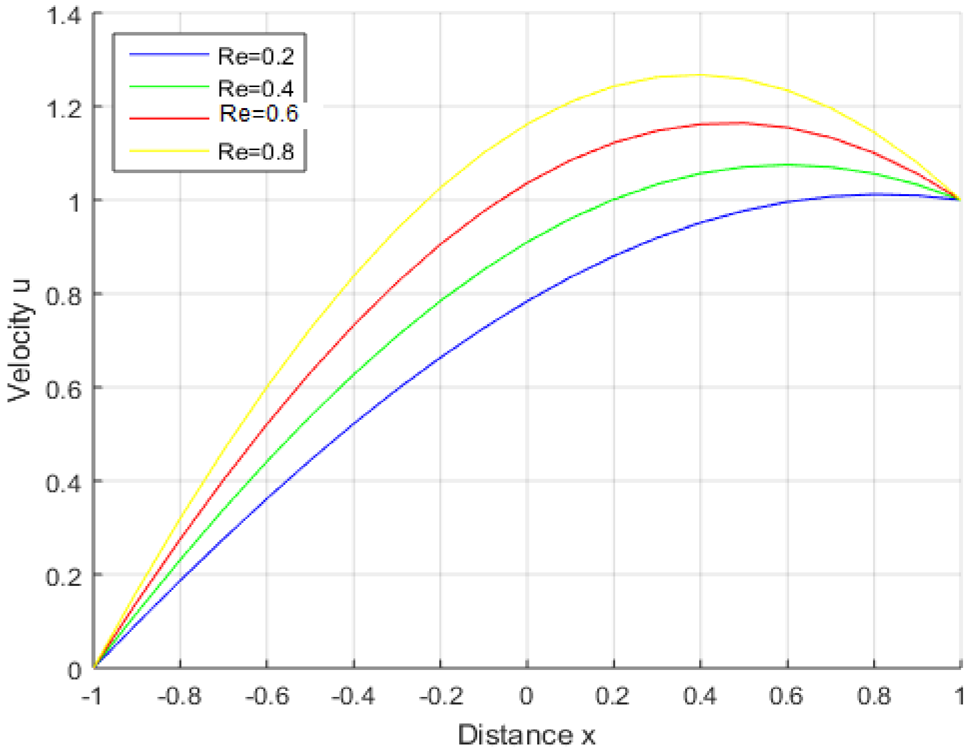

Velocity u
Distance x
Re=0.2
Re=0.4
Re=0.6
Re=0.8

Fig 4. 4

Fig 4.1 shows that decreasing the negative values of Hartmann number leads to an increase of the velocity profile while increasing positive value of the Hartmann number decreases the velocity of the fluid flow. The Hartmann number is the ratio of magnetic force to viscosity and we have varied the values of Hartmann number while the other parameters are fixed Q= -1, $\beta = 45^0, Re = 0.2$.

Fig 4.2 shows that when the angle of inclination is increased then the velocity profile is increased and when the angle of inclination is decreased leads to a decrease of velocity. When the angle is at 90 the magnetic field is normal. We have varied the angles of inclination while the other parameters are fixed M=1,Q= -1, $Re = 0.2$.

Fig 4.3 shows the effect of Q and it is observed from the figure, decreasing the negative values of pressure gradient and gravitational force leads to an increase of velocity profile while increasing the positive values of Q leads to a decrease of velocity profile. We have varied the values of Q while the other parameters are fixed M=1, $\beta = 45^0, Re = 0.2$.

Fig 4.4 shows that increasing the Reynolds number leads to increase of the velocity profile while decreasing the values of Reynolds number leads to a decrease of the velocity profile because the Reynolds number is the ratio of inertia force to viscous force. We have varied the values of Reynolds number while the other parameters are fixed M=1, Q= -1, $\beta = 45^0$.

V. CONCLUSION

The investigation on the steady incompressible viscous fluid flow through vertical plates under the presence of inclined magnetic field has been done. The fluid flow is initially at rest and one plate is stationary and the other one is moving. From the graphs we have observed that decreasing the negative values of Hartmann leads to increasing of velocity of the fluid flow while increasing the values of Hartmann number decreases the velocity distribution. The results also show that increasing the angle of inclination leads to an increase of velocity distribution. The effect of Q (pressure gradient and gravitational force) is observed in the way that when the negative values of these forces are decreased then the velocity profile will increase while increasing the positive values of pressure gradient and gravitational force leads to decrease of velocity profile. The Reynolds number is the ratio of inertia force to viscous force, the results shows that increasing Reynolds number leads to an increase of velocity distribution while decreasing Reynolds number leads to a decrease of velocity distribution. These results are of the great importance in solving space vehicle propulsion. Power convention: the extraction of electrical energy from MHD is very interesting problem. It is so-called magneto hydrodynamic.

Investigation on the unsteady MHD flow case can be carried out; the investigation can be done on the fluid flow with heat and mass transfer. We recommend that further research can be done for the same problem including the chemical reaction with radiation.

REFERENCES

[1] Faraday .M. Experimental researches in Electricity, Phil. Trans, Vol.15, 1832, p.175

[2] Ritchie. Experimental Researches in voltaic Electricity and Electromagnetism, Phil. Trans. Roy Soc. London Vol. 1832,122.279.

[3] Soundalgekar V M, Gupta S K and Aranake RN. Free convection effects on MHD stokes problem for a vertical plate Nuclear Engineering and Design (5(3),1979,313-407

[4] Raptis A and Sing A K . MHD free convection flow past accelerated vertical plate. International Communication in Heat and Mass Transfer. 10(4) ,1983,313-321,

[5] Helmy KA .MHD Unsteady free Convection past a Vertical Porous Plate .ZAMM- Journal of Applied Mathematics and Mechanics 78(4) ,1998,255-270

[6] Hazarika, Gc, And Sarma, U. Effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence of a magnetic field. Lat. Am. J. Phys. Educ. 5(1),2011, pp 100-106

[7] R. Lakshmi 2014.The numerical solution of MHD flow over a moving vertical porous plate with heat and mass transfer. Int.J.Chem.Sci. 12(4),2014, 1487-1499.

[8] Nor Raiham Mohamad Asimoni et al.MHD free convective fluid of incompressible viscous fluid past a vertical plate in the presence of magnetic field. Journal of Physics, Phys: conf.Ser.890 012009, 2017

[9] Mburu. A., Onyango .T , Kwanza. Investigation of MHD flow between two parallel infinite plates subjected to inclined magnetic field under applied pressure gradient. (Journal of Multidisciplinary Science and Technology.ISSN2455.9403, vol.3, 2016

[10] Thiele, M. Stationary transverse diffusion in horizontal porous medium boundary-layer flow with concentration-dependent fluid density and viscosity. Transport in porous media, Vol. 36,1999, pp.341-355