Zagreb indices of Hanoi graph and its complement

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ABSTRACT: *In this paper, we obtained some new properties of Zagreb indices. We mainly give explicit formulae for the first Zagreb index(coindex) Hanoi graph and its complement.*

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I. Introduction

Let $G=(V,E)$ be a graph. The number of vertices of G we denote by n and the number of edges we denote by m, thus $|V(G)|=n$ and $|E(G)|=m$. denoted by uv. The degree of a vertex $v \in V(G)$ (= number of vertices adjacent to v) is denoted by $d_G(v)$. For undefined terminologies we refer the reader to [2].

A graph invariant is any function on a graph that does not depend on a labeling of its vertices. Such quantities are also called topological indices. Hundreds of different invariants have been employed to date (with unequal success) in QSAR/QSPR studies. Among more useful of them appear two that are known under various names, but mostly as Zagreb indices. There are two invariants called the first Zagreb index and second Zagreb index [1], defined as

$$M_1(G)=\sum_{u\in V(G)} d_G(u)^2 \bmod 10mm and \bmod 10mm M_2(G)=\sum_{uv\in E(G)} d_G(u)d_G(v),$$

respectively.

In fact, one can rewrite the first Zagreb index as

$$M_1(G)=\sum_{uv\in E(G)} [d_G(u)+d_G(v)].$$

Noticing that contribution of nonadjacent vertex pairs should be taken into account when computing the weighted Winer polynomials of certain composite graphs defined first Zagreb coindex and second Zagreb coindex as

$$\overline{M_1}(G)=\sum_{uv\notin E(G)} [d_G(u)+d_G(v)] \bmod 10mm and \bmod 10mm \overline{M_2}(G)=\sum_{uv\notin E(G)} d_G(u)d_G(v),$$

respectively.

Hanoi Graph H_n

The tower of Hanoi puzzle invented in 1883 by the French Mathematician E. Lucas, has became a classic example in the analysis of algorithms and discrete mathematics [3]. he puzzle consists of n discs, no two of the same size, stacked on three vertical pegs, in such a way that no disc lies on top of a smaller disc. A permissible move is to take a top disc from one of the pegs and move it to one of the other pegs as long as it is not placed on top of a smaller disc. he set of configurations of the puzzle, together with the permissible moves, thus forms a graph in a natural way.

The Hanoi graph H_n can be constructed by taking the vertices to be the odd binomial coefficients of the Pascal triangle, computed on the integers from 0 to $2^n - 1$ and drawing an edge whenever the coefficients are adjacent diagonally or horizontally [4].

In this paper, we obtained some new properties of Zagreb coindices. We mainly give explicit formulae for the first Zagreb index(coindex) of Hanoi graph and its complement.

II. Results

We begin with the following straightforward observations.

Observation 1 *By the construction of Hanoi graph it is clear that* $V(H_n) = 3n$ *and* $E(H_n) = \frac{3(3^n - 1)}{2}$. *and* $E(\overline{H_n}) = \frac{(3^n - 1)(3^n - 3)}{2}$.

Lemma 2 [1] *Let* G *be any nontrivial graph of order* n *and size* m. *Then*

$$M_1(G) + \overline{M_1}(G) = 2m(n-1).$$

Lemma 3 [1] *Let* G *be any nontrivial graph of order* n *and size* m. *Then*

$$M_1(\overline{G}) = M_1(G) + n(n-1)^2 - 4m(n-1).$$

Theorem 4 *For all positive integers* n, $M_1(H_n) = 3^{n+2} - 15$.

Proof. Since H_n has 3^n vertices. Therefore, $\sum_{v \in V(H_n)} (v)^2 = \sum_{v \in V(H_n)} (v)^2$

By the construction of H_n it is clear that there exists exactly three vertices of degree 2 and remaining all vertices having degree 3. Therefore, $\sum_{v \in V(H_n)} (v)^2 = 12 + (3^n - 3)9 = 3^{n+2} - 15$.

This completes the proof.

Corollary 5 *For all positive integers* n, $\overline{M_1}(H_n) = 3[(3^n - 1)^2 - 3^{n+2} + 5]$

Proof. Apply Lemma 2 and Theorem 4, bearing in mind that $V(H_n) = 3n$ and

$$E(H_n) = \frac{3(3^n - 1)}{2}.$$

Theorem 6 *For all positive integers* n, $M_1(\overline{H_n}) = 3[3^{n+1} - 5] + (3^n - 1)^2(3^n - 6)$.

Proof. Apply Lemma 3 and Theorem 4, bearing in mind that $V(H_n) = 3n$ and

$$E(H_n) = \frac{3(3^n - 1)}{2}.$$

Corollary 7 *For any graph* G *of order* n *and size* m. *Then*

$$\overline{M_1}(\overline{H_n}) = 3[(3^n - 1)^2 - (3^{n+1} - 5)]$$

Proof. Apply Lemma 2 and Theorem 6, bearing in mind that $V(\overline{H_n}) = 3n$ and

$$E(\overline{H_n}) = \frac{(3^n - 1)(3^n - 3)}{2}.$$

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