Analytical Approximations of Real-Time Systems with Separate Queues to Channels and Preemptive Priorities (Worst Case)

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***ABSTRACT**: We consider a real-time multi-server system with identical servers (such as machine controllers, unmanned aerial vehicles,overhearing devices, etc.) which can be adjusted/programmed for different types of activities (e.g. active or passive). This system provides a service for real-time jobs arriving via several channels (such as assembly lines, surveillance regions, communication channels, etc.) and involves maintenance. We perform the worst case analysis of the system working under maximum load with preemptive priorities assigned for servers of different activity type. We consider a system with separate queue to each channel. Two models with ample maintenance teams and shortage of maintenance teams are treated. We provide analytical approximations of steady state probabilities for these real-time systems and check their quality.*

***KEYWORDS** -analytical approximations, preemptive priority, queue, real-time system.*

I. Introduction

Real-time systems (RTS) are imbedded in modern technological structures, such as production control systems, telecommunications systems, self-guided missiles, radars, aircraft, reconnaissance, etc.

According to Stankovic [1]: "*real time systems are thosesystems that produce results in a timely manner*", i.e. an action performed out of time constraints (too early or too late) may be useless, or harmful – even if such an action or computation is functionally correct.

Input and output of RTS are mostly uncertain or incomplete. Many of RTS include therefore stochastic as well as dynamic components.

There exists a rich literature covering various real time models.

We will focus on RTS with a *zero deadline for the beginning of job processing*. In these systems, jobs are processed immediately upon arrival, conditional on system availability. That part of the job which is not processed immediately is lost forever, and queueing of jobs (or their parts) in these systems is impossible.

The following works study this type of RTS. Kreimer and Mehrez ([2], [3]) proved that the non-mix policy maximizes the availability of a multiserver single-channel RTS involving preventive maintenance and working in general regime with any arrival pattern under consideration and constant service and maintenance times. In [4] and [5] multi-server and multi-channel RTS (with ample and limited number of maintenance facilities respectively), working under maximum load regime were treated as finite source queues ([6]). In [7] various performance measures for RTS with arbitrary number of channels operating under a maximum load regime were studied. In [8], [9] and [10] multi-server and multi-channel RTS working in *general regime* were presented.

In [11] we have shown that even very large number of servers in RTS with ample maintenance facilities does not guarantee the maximum system availability, and optimal routing probabilities were computed analytically (for exponentially distributed service times) and via Cross Entropy (CE) [12],[13], [14] simulation approach (for generally distributed service times). These results were extended for RTS with limited maintenance facilities in [15].

RTS with priorities were studied in [16], [17], [18] (preemptive) and [19] (non-preemptive) correspondingly. In [16] a multi-server and multi-channel RTS with separate queues *of servers* for each channel and preemptive priorities was studied and a set of balance linear equations for steady-state probabilities was obtained. Unfortunately, these equations do not have analytical product-form solutions.

In [19] several approximations methods, using the modifications of techniques proposed in [20], [21]were tested, and the best one was chosen.

The work presented here provides analytical approximations (based on the best method proposed in [18]) of steady state probabilities for RTS with separate queues *of servers* for each channel and preemptive priorities. We compare the approximation results with exact values of steady state probabilities and system performances.

The paper is organized as follows: In Section 2, the description of the model with preemptive priorities is presented. In Section 3 balance equations (see [16]) for models with ample maintenance teams and shortage of maintenance teams are presented. Section 4 provides analytical approximations for these models. In Section 5 some numerical results are presented. Finally, Section 6 is devoted to conclusions.

II. Description of the model

The most important characteristics of RTS with a zero deadline for the beginning of job processing are summarized in [8]. A real-world problem was studied in [16]. Here we provide the formal description of RTS from [16].

The system consists of r identical *channels*. For proper performance each channel needs exactly one fixed server at *any* moment (worst case), otherwise the information in this channel (at this specific moment) is lost. There are *N servers* (which are subject to breakdowns) in the system. A server, which is out of order, needs R_i time units of maintenance. After repair a fixed server may be of *u*-th type of *quality*(*u=1,…,m*) and is assigned to the *v*-th channel with probability $p_{u,v}$, (*u=1,…,m;v=1,…,r*). These probabilities can be used as control parameters. Only *after* the repair is completed, the quality control procedure determines the quality type of fixed server. The fixed server of*u*-th type assigned to *v*-th channel is operative for a period of time $S_{u,v}$ before requiring R_i hours of repair. $S_{u,v}$ and R_i areindependent exponentially distributed random values with parameters $\mu_{u,v}$ (*u=1,…,m; v=1,…,r*) and λ respectively. It is assumed that there are K identical maintenance teams in the system. Each team can repair exactly one server at a time. We will consider two models: 1) with ample maintenance facilities ($K \geq N$) so that all *N* servers can be repaired simultaneously if necessary, and 2) with shortage of maintenance facilities (*K*<*N*), in which case some of broken servers will wait for maintenance.

The duration times R_i of repair are i.r.v. exponentially distributed with parameter λ , which does not depend on the quality type of the server (neither before nor after the repair). After repair, the fixed server will either be on stand-by or operating inside the channel. There is a separate queue of servers to each channel.

We assume that servers of the first kind of quality type have the highest priority, servers of the second quality type are the next priority in line, and so on. Finally, servers of the *m*-th quality type have the lowest priority. Server operating in any channel is *interrupted*, if another fixed server of higher priority type arrives from maintenance. When the operating server must be repaired, the fixed server with highest priority takes its place.

The system works under a maximum load (worst case) of nonstop data arrival to each channel). Thus, there exist a total of exactly *r* jobs in the whole system at any moment, and the *nonstop* operation of each channel is needed.

If, during some period of time of length T ,there is no fixed server to provide the proper operation of the channel, we will say that the part of the job of length T is lost forever.

III. Balance equations for steady state probabilities

In [15] the state of the system was defined as $\begin{pmatrix} n_{1,1},\ldots,n_{m,1} \\ \vdots \\ n_{1,r},\ldots,n_{m,r} \end{pmatrix}$, where $n_{u,v}$ *u=1,…,m; v = 1,…,r* is a number of fixed servers of *u*-th activity type assigned for *v*-th channel (obviously, $\sum_{u=1}^{m}\sum_{v=1}^{r} n_{u,v} \leq N$ and $n_{u,v} \geq 0$), and

$P_{\begin{pmatrix} n_{1,1},\ldots,n_{m,1} \\ \vdots \\ n_{1,r},\ldots,n_{m,r} \end{pmatrix}}$ the corresponding steady state probability. There are $\begin{pmatrix} N+m\cdot r \\ m\cdot r \end{pmatrix}$ states in total. We also denote $\lambda_{u,v} = \lambda p_{u,v}$, the rate of assignments of fixed servers of u-th activity type to the v-th channel.

We will consider two models: 1) with ample maintenance facilities ($K \geq N$); and 2) with shortage of maintenance facilities ($K<N$).

We will focus on the case $r < N$, otherwise all fixed servers will be busy and preemptive priority regime, therefore, will not work.

3.1 Model with ample maintenance facilities

We assume that there are ample identical maintenance facilities $K \geq N$ available to repair all N servers simultaneously, if needed. Thus, each broken server enter maintenance facilities without delay. This model was studied, and the following set of linear equations for steady state probabilities was obtained in [16].

$$P_{\begin{pmatrix} n_{1,1},\ldots,n_{m,1} \\ \vdots \\ n_{1,v},\ldots,n_{u,v},\ldots,n_{m,v} \\ \vdots \\ n_{1,r},\ldots,n_{m,r} \end{pmatrix}} \left[\left(N - \sum_{v=1}^{r}\sum_{i=1}^{m} n_{i,v} \right)\lambda + \sum_{v=1}^{r}\sum_{t=1}^{m} \min\left(n_{t,v}, \max\left(0, 1-\delta_t \sum_{i=1}^{t-1} n_{i,v} \right)\right)\mu_{t,v} \right] =$$

$$= \sum_{v=1}^{r}\sum_{u=1}^{m} \min\left(n_{u,v}, 1\right)\cdot P_{\begin{pmatrix} n_{1,1},\ldots,n_{m,1} \\ \vdots \\ n_{1,v},\ldots,n_{u,v}-1,\ldots,n_{m,v} \\ \vdots \\ n_{1,r},\ldots,n_{m,r} \end{pmatrix}} \left(N+1-\sum_{v=1}^{r}\sum_{i=1}^{m} n_{i,v} \right)\lambda_{u,v} +$$

$$+ \min\left(N - \sum_{v=1}^{r}\sum_{i=1}^{m} n_{i,v}, 1 \right)\cdot \sum_{v=1}^{r}\sum_{u=1}^{m} P_{\begin{pmatrix} n_{1,1},\ldots,n_{m,1} \\ \vdots \\ n_{1,v},\ldots,n_{u,v}+1,\ldots,n_{m,v} \\ \vdots \\ n_{1,r},\ldots,n_{m,r} \end{pmatrix}} \cdot \min\left(n_{u,v}+1, \max\left(0, 1-\delta_u \sum_{i=1}^{u-1} n_{i,v} \right)\right)\mu_{u,v} \quad (1)$$

$$\sum_{v=1}^{r}\sum_{n_{1,1}=0}^{N}\sum_{n_{2,1}=0}^{N-n_{1,1}} \cdots \sum_{n_{m,v}=0}^{N-\sum_{i=1}^{m-1} n_{i,v}} P_{\begin{pmatrix} n_{1,1},\cdots,n_{m,1} \\ \vdots \\ n_{1,v},\cdots,n_{m,v} \\ \vdots \\ n_{1,r},\cdots,n_{m,r} \end{pmatrix}} = 1, \quad (2)$$

where $\lambda = \sum_{v=1}^{r}\sum_{u=1}^{m} \lambda_{u,v}; \delta_t = 1-\delta_{t1}; \delta_{t1} = \begin{cases} 0, t \neq 1 \\ 1, t = 1 \end{cases}; \delta_u = 1-\delta_{u1}; \delta_{u1} = \begin{cases} 0, u \neq 1 \\ 1, u = 1 \end{cases}.$

3.2 Model with shortage of maintenance facilities

Here we assume that there are K ($K < N$) maintenance facilities in the system. Thus, a shortage of maintenance facilities is possible when there are more than K broken servers in the system. In that case, the broken server waits for maintenance. This model was studied in [16], and the following set of linear balance equations for steady state probabilities was obtained.

$$P_{\begin{pmatrix} n_{1,1},\dots,n_{m,1} \\ \vdots \\ n_{1,v},\dots,n_{u,v},\dots,n_{m,v} \\ \vdots \\ n_{1,r},\dots,n_{m,r} \end{pmatrix}} \left[\min\left(K, N-\sum_{v=1}^{r}\sum_{i=1}^{m} n_{i,v}, K\right)\lambda + \sum_{v=1}^{r}\sum_{t=1}^{m} \min\left(n_{t,v}, \max\left(0, 1-\delta_t \sum_{i=1}^{t-1} n_{i,v}\right)\right)\mu_{t,v} \right] =$$

$$= \sum_{v=1}^{r}\sum_{u=1}^{m} \min(n_{u,v},1)\cdot P_{\begin{pmatrix} n_{1,1},\dots,n_{m,1} \\ \vdots \\ n_{1,v},\dots,n_{u,v}-1,\dots,n_{m,v} \\ \vdots \\ n_{1,r},\dots,n_{m,r} \end{pmatrix}} \min\left(K, N+1-\sum_{v=1}^{r}\sum_{i=1}^{m} n_{i,v}, K\right)\lambda_{u,v} +$$

$$+ \min\left(N-\sum_{v=1}^{r}\sum_{i=1}^{m} n_{i,v}, 1\right)\cdot \sum_{v=1}^{r}\sum_{u=1}^{m} P_{\begin{pmatrix} n_{1,1},\dots,n_{m,1} \\ \vdots \\ n_{1,v},\dots,n_{u,v}+1,\dots,n_{m,v} \\ \vdots \\ n_{1,r},\dots,n_{m,r} \end{pmatrix}} \cdot \min\left(n_{u,v}+1, \max\left(0, 1-\delta_u \sum_{i=1}^{u-1} n_{i,v}\right)\right)\mu_{u,v} \qquad (3)$$

$$\sum_{v=1}^{r}\sum_{n_{1,1}=0}^{N}\sum_{n_{2,1}=0}^{N-n_{1,1}} \dots \sum_{n_{m,v}=0}^{N-\sum_{i=1}^{m-1} n_{i,v}} P_{\begin{pmatrix} n_{1,1},\cdots,n_{m,1} \\ \vdots \\ n_{1,v},\cdots,n_{m,v} \\ \vdots \\ n_{1,r},\cdots,n_{m,r} \end{pmatrix}} = 1 \qquad (4)$$

IV. Approximations

We apply the Method 3 developed in [18] to get analytical approximations for solutions of equations (1)-(2). This method is a combination of the methods presented in [20] and [21]. These approximations instead of *Global Balance Equations* ((1)-(2) or (3)-(4)) use *Local Balance Equations*, and provide analytical *product form solutions* (see [6]) for steady state probabilities. We will use an RSS (root of sum of squares) criteria in order to evaluate the quality of these approximations:

$$RSS = SQRT\{\sum_{v=1}^{r}\sum_{n_{1,1}=0}^{N}\sum_{n_{2,1}=0}^{N-n_{1,1}} \dots \sum_{n_{m,v}=0}^{N-\sum_{i=1}^{m-1} n_{i,v}} [P_{\begin{pmatrix} n_{1,1},\dots,n_{m,1} \\ \vdots \\ n_{1,v},\dots,n_{m,v} \\ \vdots \\ n_{1,r},\dots,n_{m,r} \end{pmatrix}} - P'_{\begin{pmatrix} n_{1,1},\dots,n_{m,1} \\ \vdots \\ n_{1,v},\dots,n_{m,v} \\ \vdots \\ n_{1,r},\dots,n_{m,v} \end{pmatrix}}]^2\}, \qquad (5)$$

where $P_{\begin{pmatrix} n_{1,1},\dots,n_{m,1} \\ \vdots \\ n_{1,r},\dots,n_{m,r} \end{pmatrix}}$ and $P'_{\begin{pmatrix} n_{1,1},\dots,n_{m,1} \\ \vdots \\ n_{1,r},\dots,n_{m,r} \end{pmatrix}}$ are exact numerical solution (of equations (1)-(2) or (3)-(4)) and analytical approximation respectively.

Then analytical approximations of steady state probabilities for the model with ample maintenance teams and separate queues of servers to each channel are as follows.

$$P'_{\begin{pmatrix} n_{1,1},\dots,n_{m,1} \\ \cdot \\ \cdot \\ \cdot \\ n_{1,r},\dots,n_{m,r} \end{pmatrix}} = \frac{\left(\prod_{j=0}^{\left(\sum_{v=1}^{r}\sum_{u=1}^{m} n_{u,v}\right)-1} (N-j) \right) \left(\prod_{v=1}^{r}\prod_{u=1}^{m} \lambda_{u,v}^{n_{u,v}} \right)}{\prod_{v=1}^{r} \left(\mu_{m,v}^{n_{m,v}} \cdot \prod_{u=m-1}^{1} \left(\left(\prod_{j=\sum_{i=m}^{(u+1)} n_{i,v}+1}^{\sum_{i=m}^{u} n_{i,v}} (\mu_{u,v} + (N-j)\lambda_{m,v}) \right) + \delta_{u,v} \right) \right)} P'_{\begin{pmatrix} 0,\dots0 \\ \vdots \\ 0,\dots,0 \end{pmatrix}} \quad (6)$$

where $\delta_{u,v} = \begin{cases} 1, n_{u,v} = 0 \\ 0, n_{u,v} > 0 \end{cases}$ and

$$\sum_{v=1}^{r} \sum_{n_{1,1}=0}^{N} \sum_{n_{2,1}=0}^{N-n_{1,1}} \dots \sum_{n_{m,v}=0}^{N-\sum_{i=1}^{m-1} n_{i,v}} P'_{\begin{pmatrix} n_{1,1},\cdots,n_{m,1} \\ \vdots \\ n_{1,v},\cdots,n_{m,v} \\ \vdots \\ n_{1,r},\cdots,n_{m,r} \end{pmatrix}} = 1$$

$$P'_{\begin{pmatrix} 0,\dots,0 \\ \cdot \\ \cdot \\ \cdot \\ 0,\dots,0 \end{pmatrix}} = [1 + \sum_{v=1}^{r} \sum_{n_{1,1}=0}^{N} \dots \sum_{n_{m,v}=0}^{N-\sum_{i=1}^{m-1} n_{i,v}} \frac{\left(\prod_{j=0}^{\left(\sum_{v=1}^{r}\sum_{u=1}^{m} n_{u,v}\right)-1} (N-j) \right) \left(\prod_{v=1}^{r}\prod_{u=1}^{m} \lambda_{u,v}^{n_{u,v}} \right)}{\prod_{v=1}^{r} \left(\mu_{m,v}^{n_{m,v}} \cdot \prod_{u=m-1}^{1} \left(\left(\prod_{j=\sum_{i=m}^{(u+1)} n_{i,v}+1}^{\sum_{i=m}^{u} n_{i,v}} (\mu_{u,v} + (N-j)\lambda_{m,v}) \right) + \delta_{u,v} \right) \right)}]^{-1}$$

The corresponding analytical approximations of steady state probabilities for the model with shortage of maintenance teams and separate queues of servers to each channel are as follows.

$$P'_{\begin{pmatrix} n_{11},\dots,n_{m1} \\ \cdot \\ \cdot \\ \cdot \\ n_{1r},\dots,n_{mr} \end{pmatrix}} = \begin{Bmatrix} if \sum_{v=1}^{r}\sum_{u=1}^{m} n_{u,v} \le N-K : \\ \dfrac{K^{\sum_{v=1}^{r}\sum_{u=1}^{m} n_{u,v}} (\prod_{v=1}^{r}\prod_{u=1}^{m} \lambda_{u,v}^{n_{u,v}})}{\prod_{v=1}^{r} \left({\mu_{m,v}}^{n_{m,v}} \cdot \prod_{u=m-1}^{1} (\mu_{u,v} + K\lambda_{m,v})^{n_{u,v}} \right)} P'_{\begin{pmatrix} 0,\dots,0 \\ \vdots \\ 0,\dots,0 \end{pmatrix}} \\ if \sum_{v=1}^{r}\sum_{u=1}^{m} n_{uv} > N-K : \\ \dfrac{K^{N-K} \left(\prod_{u=N-K}^{\sum_{v=1}^{r}\sum_{i=1}^{m} n_{i,v}-1} (N-u) \right) (\prod_{v=1}^{r}\prod_{u=1}^{m} \lambda_{u,v}^{n_{u,v}})}{\prod_{v=1}^{r} \left(\mu_{m,v}^{n_{m,v}} \cdot \prod_{u=m-1}^{1} \left(\mu_{u,v} + K\lambda_{m,v} \right)^{n_k} \cdot \left(\left(\prod_{j=\max(N-K, \sum_{l=1}^{v-1}\sum_{i=m}^{1} n_{i,l} + \sum_{i=m}^{u+1} n_{i,v})+1}^{\sum_{l=1}^{v}\sum_{i=m}^{u} n_{i,l}} (\mu_{u,v} + (N-j)\lambda_{m,v}) \right) + \delta_{u,v} \right) \right)} P'_{\begin{pmatrix} 0,\dots,0 \\ \vdots \\ 0,\dots,0 \end{pmatrix}} \end{Bmatrix}$$

where $n_k = \max\left[\min\left(N-K-\sum_{l=1}^{r}\sum_{i=m}^{u+1} n_{i,l}, n_{u,v}\right), 0\right]$; $\delta_{u,v} = \begin{Bmatrix} 1, n_{u,v} = 0 \\ 0, n_{u,v} > 0 \end{Bmatrix}$ and

$$\sum_{v=1}^{r}\sum_{n_{1,1}=0}^{N}\sum_{n_{2,1}=0}^{N-n_{1,1}}\cdots\sum_{n_{m,v}=0}^{N-\sum_{i=1}^{m-1} n_{i,v}} P'_{\begin{pmatrix} n_{1,1},\cdots,n_{m,1} \\ \vdots \\ n_{1,v},\cdots,n_{m,v} \\ \vdots \\ n_{1,r},\cdots,n_{m,r} \end{pmatrix}} = 1$$, from which the value of $P'_{\begin{pmatrix} 0,\ldots,0 \\ \vdots \\ 0,\ldots,0 \end{pmatrix}}$ can be easily found.

V. Numerical results

In this Section, we present some numerical results, showing the quality of our approximations in terms of RSS. Table 1 contains *exact values* of steady state probabilities for the model with ample maintenance facilities obtained from equations (1)-(2) presented in [16].

We consider the model with following parameters $r = 2$; $N = 3$; $\lambda = 28$; $m = 2$; $\mu_{11} = 8$, $\mu_{21} = 10$, $\mu_{12} = 6$, $\mu_{22} = 12$; $p_{11} = 0.21$, $p_{21} = 0.29$, $p_{12} = 0.14$, $p_{22} = 0.36$; $K \geq N$.

Table 1. Steady state probabilities - exact numerical solution of equations (1)-(2)

n_{11},n_{21} n_{12},n_{22}	*P(n)*	n_{11},n_{21} n_{12},n_{22}	*P(n)*	n_{11},n_{21} n_{12},n_{22}	*P(n)*	n_{11},n_{21} n_{12},n_{22}	*P(n)*
0,0 0,0	0.008	0,0 3,0	0.003	1,0 2,0	0.003	1,1 0,1	0.045
0,0 0,1	0.027	1,0 0,0	0.009	0,1 0,2	0.044	1,1 1,0	0.021
0,0 1,0	0.005	0,1 0,0	0.028	0,1 1,1	0.038	0,2 0,1	0.046
0,0 0,2	0.058	1,0 0,1	0.024	0,1 2,0	0.007	0,2 1,0	0.024
0,0 1,1	0.029	1,0 1,0	0.005	2,0 0,0	0.009	3,0 0,0	0.007
0,0 2,0	0.004	0,1 0,1	0.051	1,1 0,0	0.039	2,1 0,0	0.034
0,0 0,3	0.048	0,1 1,0	0.017	0,2 0,0	0.060	1,2 0,0	0.084
0,0 1,2	0.087	1,0 0,2	0.030	2,0 0,1	0.012	0,3 0,0	0.048
0,0 2,1	0.027	1,0 1,1	0.016	2,0 1,0	0.005		

Table 2 contains their analytical approximations counterparts for the same parameters: $r = 2$; $N = 3$; $\lambda = 28$; $m = 2$; $\mu_{11} = 8$, $\mu_{21} = 10$, $\mu_{12} = 6$, $\mu_{22} = 12$; $p_{11} = 0.21$, $p_{21} = 0.29$, $p_{12} = 0.14$, $p_{22} = 0.36$; $K \geq N$.

Table 2.Steady state probabilities – analytical approximations.

n_{11},n_{21} n_{12},n_{22}	$P'(n)$	n_{11},n_{21} n_{12},n_{22}	$P'(n)$	n_{11},n_{21} n_{12},n_{22}	$P'(n)$	n_{11},n_{21} n_{12},n_{22}	$P'(n)$
0,0 0,0	0.016	0,0 3,0	0.005	1,0 2,0	0.004	1,1 0,1	0.021
0,0 0,1	0.040	1,0 0,0	0.012	0,1 0,2	0.053	1,1 1,0	0.017
0,0 1,0	0.007	0,1 0,0	0.038	0,1 1,1	0.043	0,2 0,1	0.051
0,0 0,2	0.067	1,0 0,1	0.020	0,1 2,0	0.013	0,2 1,0	0.041
0,0 1,1	0.020	1,0 1,0	0.006	2,0 0,0	0.009	3,0 0,0	0.007
0,0 2,0	0.004	0,1 0,1	0.064	1,1 0,0	0.026	2,1 0,0	0.019
0,0 0,3	0.056	0,1 1,0	0.019	0,2 0,0	0.061	1,2 0,0	0.046
0,0 1,2	0.111	1,0 0,2	0.017	2,0 0,1	0.012	0,3 0,0	0.049
0,0 2,1	0.013	1,0 1,1	0.013	2,0 1,0	0.004		

Corresponding RSSE=**0.068** (between results of Table1 and Table 2)

Table 3 contains exact numerical values as well as analytical approximations of several Performance characteristics (averages) for the same model: $r = 2$; $N = 3$; $\lambda = 28$; $m = 2$; $\mu_{11} = 8$, $\mu_{21} = 10$, $\mu_{12} = 6$, $\mu_{22} = 12$; $p_{11} = 0.21$, $p_{21} = 0.29$, $p_{12} = 0.14$, $p_{22} = 0.36$; $K \geq N$.

Table 3. Exact values and analytical approximations of Performance characteristics (**averages**)

Performance characteristic	**Exact Value**	**Analytical Approximations**
Nu. of fixed servers L	2.542	2.461
Nu. of fixed serversat channel 1 $N_{(1)}$	1.308	1.142
Nu. of fixed serversat channel 2 $N_{(2)}$	1.233	1.319
Nu. of fixed serversof type 1 N_1	0.748	0.646
Nu. of fixed serversof type 2 N_2	1.793	1.815
Nu. of fixed serversof type 1 at channel 1 $L_{1,1} + Q_{1,1}$	0.412	0.282
Nu. of fixed serversof type 2 at channel 1 $L_{2,1} + Q_{2,1}$	0.896	0.860
Nu. of fixed serversof type 1 at channel 2 $L_{1,2} + Q_{1,2}$	0.336	0.364

Nu. of fixed serversof type 2 at channel 2 $L_{2,2}+Q_{2,2}$	0.897	0.955
Availability at channel 1 Av_1	0.725	0.661
Availability at channel 2 Av_2	0.650	0.696
Availability of the system Av	0.688	0.688

VI. Conclusions

We have found a good analytical approximations for RTS with a separate queues for each channel, and preemptive priorities (for servers of different quality type) working under maximal load. Two models with ample maintenance teams and shortage of maintenance teams were treated and numerical results showing the quality of approximations are provided.

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